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Numerical Analysis and Applications of Coupled Vibration in Asymmetric Laminated Composite Beams

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Abstract

This paper presents a comprehensive numerical analysis and application of extension-bending-shear-torsion coupled vibration in asymmetric laminated composite beams, in which the exact closed-form solutions were previously obtained to investigate the steady-state dynamic response of such beams under various harmonic loading and boundary conditions. Key objectives include predicting natural frequencies and corresponding mode shapes, as well as establishing quasi-static responses for different harmonic excitation frequencies. Numerical examples are used to validate the accuracy and efficiency of the proposed solutions by comparing static and dynamic results with those from existing literature and finite element analysis. The study reveals excellent agreement between the closed-form solutions and previously published results, emphasizing the utility of the developed approach for both static and dynamic coupled vibration analysis in asymmetric laminated beams. The results offer valuable insights into the dynamic behavior of asymmetric composite beams, with potential applications in engineering structures subjected to complex loading conditions.

Keywords: Coupled vibration, steady state response, asymmetric laminated beam, harmonic loading.

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التحليل العددي وتطبيقات الاهتزاز المزدوج في العارضات ذات الرقائق المركبة الغير متماثلة

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الخلاصة

يقدم هذا البحث تحليلاً عددياً شاملاً وتطبيقاً للاهتزاز المقترن بالتمدد والانحناء والقص والالتواء في العارضات ذات الرقائق المركبة الغير متماثلة، حيث تم الحصول مسبقًا على الحلول ذات الشكل المغلق الدقيق لدراسة الاستجابة الديناميكية للحالة المستقرة لهذه العارضات في ظل ظروف التحميل والحدود التوافقية المختلفة. تشمل الأهداف الرئيسية التنبؤ بالترددات الطبيعية وأشكال الأوضاع المقابلة، بالإضافة إلى إنشاء استجابات شبه ثابتة لترددات الإثارة التوافقية المختلفة. تم استخدام الأمثلة العددية للتحقق من دقة وكفاءة الحلول المقترحة من خلال مقارنة النتائج الثابتة والديناميكية مع تلك النتائج المنشورة سابقا الموجودة في الأدبيات وتحليل العناصر المحدودة. تكشف الدراسة عن اتفاق ممتاز بين الحلول ذات الشكل المغلق الدقيق والنتائج المنشورة مسبقًا، مع التركيز على فائدة النهج المطور لكل من تحليل الاهتزاز الثابت والديناميكي في العارضات المصفحة غير المتماثلة. تقدم النتائج رؤى قيمة حول السلوك الديناميكي للعارضات المركبة غير المتماثلة، مع التطبيقات المحتملة في الهياكل الهندسية المعرضة لظروف التحميل المعقدة.

الكلمات المفتاحية: الاهتزاز المقترن، استجابة الحالة المستقرة، شعاع رقائقي غير متماثل، التحميل التوافقي.

1. Introduction and Objective

Composite laminated beams are essential in modern engineering applications due to their superior mechanical properties, such as high strength-to-weight and stiffness-to-weight ratios. These materials, used in aerospace, automotive, and civil engineering, often face complex dynamic loading conditions, including axial, bending, shear, and torsional excitations. Understanding the dynamic behavior of such beams, especially those with asymmetric laminations, is important for ensuring the structural integrity of these systems under operational loads.

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Previous research has focused on free vibration analyses and linear responses of composite beams, with a few studies addressing the coupled dynamic behavior under harmonic excitations.In previous published research [Nagiar and Hjaji 2024] on dynamic analysis of coupled axial-bending-shear-torsion responses in asymmetric laminated beams, closed-form solutions were derived for different harmonic excitations and boundary conditions, providing insight into the quasi-static and dynamic behaviors of such systems. These formulations were based on first-order shear deformation theory (*FSDT*) and incorporated the effects of rotary inertia, Poisson's ratio, and material anisotropy, which are critical in capturing the complex interactions between axial, bending, and torsional deformations.The governing field equations for these composite beams, derived using Hamilton's variational principle, were solved analytically for steady-state responses under harmonic excitations. The focus was on determining the natural frequencies, mode shapes, quasi-static response steady-state dynamic behavior of the laminated beams when subjected to dynamic bending and torsional loads. These solutions provide significant insights into the design and analysis of laminated composite structures, particularly for applications in aerospace, mechanical, and civil engineering, where dynamic loads play an important role.
In this extended analysis, the exact solutions not only capture the steady-state dynamic response but also offer predictions for the quasi-static response at low excitation frequencies. Moreover, the solutions were validated against exact solutions and finite element analysis (*FEA*) models, showcasing their accuracy and effectiveness. This work lays the foundation for further exploration of more complex beam configurations, loading conditions, and potential non-linearities in future research.

This paper extends the scope of previous research of Nagiar and Hjaji (2024) by focusing on the numerical analysis and applications of these coupled vibrations in asymmetric laminated composite beams. The primary goals of this study are to verify the analytical closed-form solutions developed by Nagiar and Hjaji (2024) through numerical simulations using finite element analysis (FEA) and to investigate how different boundary conditions, excitation frequencies, and material properties influences the dynamic behavior of the laminated beams. Additionally, the study aims to predict natural frequencies and mode shapes, and to validate both

http://www.doi.org/10.62341/hama1117

the quasi-static and steady state responses under harmonic excitations against exact solutions and finite element models.

2. Numerical Examples and Discussion

In this section, two numerical examples are presented to validate the accuracy of the closed-form solutions derived in Nagiar and Hjaji (2024). These examples are provided to analyze statically and dynamically the extension-bending-shear-torsion coupled responses of asymmetric laminated beams under various harmonic bending and twisting loads, by considering different boundary conditions. In these examples, the laminates have uniform thickness and are made from the same composite material. The quasi-static results obtained from the closed-form solutions are compared with exact and finite element solutions available in the literature, while the dynamic results computed from the present exact solutions are compared with Abaqus finite shell element solution. In Abaqus model, the shell S4R element used has four nodes, each with six degrees of freedom (three translations and three rotations), and accurately captures the transverse shear deformation effects. The performance of the composite laminated beam under different harmonic excitations and boundary conditions is discussed in detail.
The closed-form solutions provided in Nagiar and Hjaji (2024) to analyze the coupled dynamic response-axial, bending, shear and torsion of asymmetric laminated beams under various harmonic excitations. The numerical results of this research achieve the following objectives:

1. Determining the steady-state dynamic response of asymmetric laminated beams when subjected to harmonic bending and twisting loads at a specific excitation frequency.
2. Capturing the quasi-static response of these beams by employing very low excitation frequencies Ωrelative to the first natural frequency of the composite beam (i.e., $\Omega \approx 0.01\omega_1$, where ω_1 represents the first natural frequency).
3. Predicting the natural frequencies and corresponding steady-state mode shapes of asymmetric laminated beams under varying harmonic excitations.

2.1 Example (1): Validation of Static and Dynamic Results

This example has been widely used by many researchers for validation purposes. To confirm the accuracy of the present closed-form solutions, a graphite-epoxy asymmetric [0^{o}/90^{o}] laminated

http://www.doi.org/10.62341/hama1117

composite beam with a span length of 0.381m and a rectangular cross-section (width$b = 25.4mm$ and thickness$h = 25.4mm$) is subjected to a uniformly distributed harmonic transverse force $q_z(x,t) = 200e^{i\Omega t}N/m$. The material properties of the laminated beam are given as (Vo and Thai 2012): $E_{11} = 25.0GPa$, $E_{22} = 1.0GPa$, $G_{12} = G_{13} = 0.5E_{22}$, $G_{23} = 0.2E_{22}$, $\upsilon_{12} = 0.25$, and $\rho = 1389.2kg/m^3$. This example wasstatically analyzed and investigated by many researchers such as Khdeir and Reddy (1994), Chakraborty et al. (2002), Murthy et al. (2005),Vo and Thai (2012),and Karkon (2020)], while the dynamic response results computed from the present exact solutions are compared with Abaqus finite shell element model solution. In Abaqus model, the given composite beam is divided into 100 S4R shell elements along the longitudinal axis of the composite beam and 4 shell S4R elements along the beam width, i.e., a total of 400 S4R shell elements with 3030 degrees of freedom are utilized to achieve the required accuracy.

2.1.1 Quasi-Static Response

The quasi-static responses for the asymmetric [$0^{o}/90^{o}$] laminated beams having different boundary conditions are analyzed across different span-to-thickness ratio denoted asL/h. Even though, the composite laminated beam subjected to transverse harmonic force $q_z(x,t) = 200e^{i\Omega t}N/m$, but the quasi-static response is captured by using very small exciting frequency of the harmonic transverse force related to the first natural frequency of the composite beam, i.e., $\Omega \approx 0.01\omega_1$, where $\omega_1$is the first natural frequency of the given composite beam.

The numerical results for static response analysis accomplished from the present exact closed-form solution are compared with the corresponding static results conducted Khdeir and Reddy (1994), Chakraborty et al. (2002), Murthy et al. (2005),Vo and Thai (2012), and Karkon (2020),while the dynamic response results computed from the present exact solutions are as illustrated in Table (1). The results for non-dimensional mid-span transverse displacement function$W_d = 100bh^3E_{22}\overline{W}(L/2)/q_zL^4$ defined in Vo and Thai (2012) are obtained at the mid-span of composite beams (i.e., $x = L/2$)for different span-to-thickness ratios$L/h = 5$,10 and 50. The influence of the span-to-thickness ratio (L/h) on the static responses for asymmetric [0°/90°] laminated composite beams having

http://www.doi.org/10.62341/hama1117

clamped-free (*CF*), simply-supported (*SS*), clamped-clamped (*CC*), and clamped-pinned(*CP*) boundary conditions are presented in Table (1). It is obvious that the static results obtained from the present closed-form solution exhibit excellent agreement with those obtained from the other solutions. As a general observation, the present closed-form solutions are successful at capturing the static response of the composite laminated beams.

Furthermore, the static results for extensional displacement $\overline{U}(x)$,transverse displacement $\overline{W}(x)$, bending rotation $\overline{\theta}(x)$ and twisting rotation angle $\overline{\Phi}(x)$are plotted along the beam axis x for a span-to-thickness ratio $(L/h) = 50$. These results are illustrated in Figures (1-4) for asymmetric cross-ply [0°/90°] laminated composite beams with clamped-free (*CF*), simply-supported (*SS*), clamped-clamped (*CC*), and clamped-pinned (*CP*) boundary conditions, respectively.

The quasi-static results based on the present closed-form solutions are compared with Abaqus finite shell element model solution. It is clearly noted that, the values of twisting angle are very small in the static results for an asymmetric cross-ply [0°/90°] laminated beam. In a $[0^o/90^o]$ cross-ply laminate, the layers are oriented at 0° and 90° to the axis of the composite beam. The 0° fibers primarily resist bending in the longitudinal direction, while the 90° fibers resist bending in the transverse direction. Since these orientations are perpendicular, the laminate exhibits high in-plane stiffness but does not inherently promote twisting coupling under pure bending. In other words, the small twisting angle of order 10^{-5} suggests that the bending-twisting coupling is not significant and remains minimal in the asymmetric cross-ply$[0^o/90^o]$ laminated beams. It is noted that, the static results obtained from the present solution provide an excellent agreement with the corresponding results based on Abaqus shell model solution. Again, the closed-form solutions are able to efficiently capture the static response of the given composite beams.

Table (1): Non-dimensional mid-span static displacement $(W_d)$of asymmetric [0°/90°] laminated beam under distributed harmonic force for different boundary conditions

L/h	**Reference**	**Non-dimensional mid-span static displacement (W_d)**			
		CF	***SS***	***CP***	***CC***
5	Khdeir and Reddy (1994)	16.436	5.036	3.320	2.379
	Chakraborty et al. (2002)	16.496	5.048	3.324	2.381
	Murthy et al. (2005)	15.334	4.750	2.855	1.924

http://www.doi.org/10.62341/hama1117

	Karkon (2020)	16.436	5.036	3.320	2.379
	Vo and Thai (2012)	16.461	5.043	-	-
	Present solution	16.450	5.037	3.322	2.379
10	Khdeir and Reddy (1994)	12.579	3.750	1.834	1.093
	Chakraborty et al. (2002)	12.607	3.751	1.835	1.094
	Murthy et al. (2005)	12.398	3.668	1.736	1.007
	Karkon (2020)	12.579	3.750	1.834	1.093
	Vo and Thai (2012)	12.369	3.757	-	-
	Present solution	12.591	3.753	1.836	1.094
50	Khdeir and Reddy (1994)	11.345	3.339	1.349	0.681
	Chakraborty et al. (2002)	11.413	3.353	1.356	0.686
	Murthy et al. (2005)	11.392	3.318	1.343	0.681
	Karkon (2020)	11.345	3.339	1.349	0.681
	Vo and Thai (2012)	11.363	3.346	-	-
	Present solution	11.361	3.342	1.350	0.682

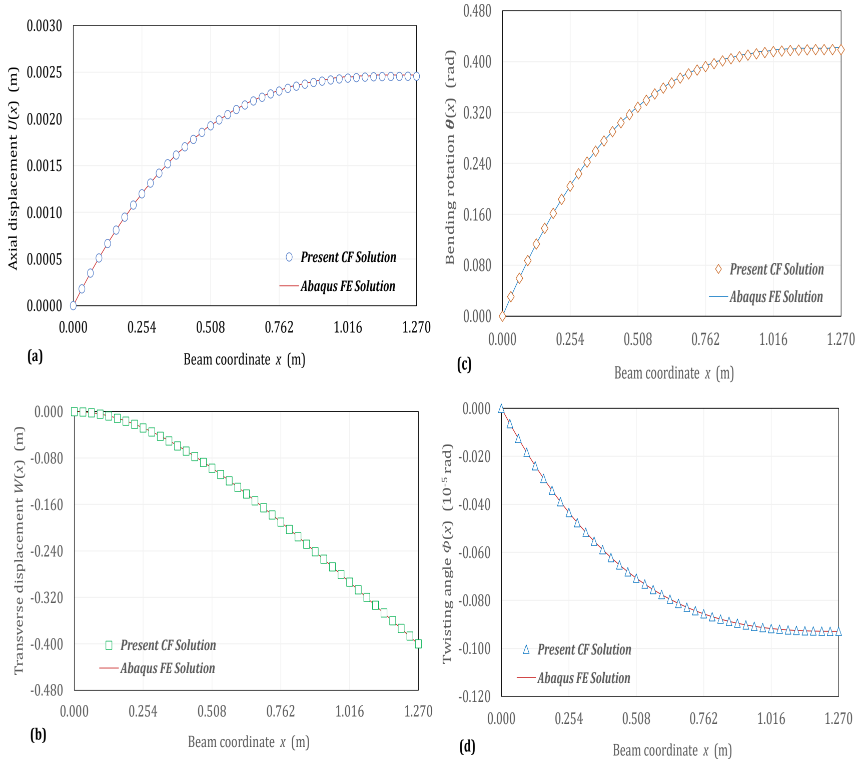

Figure 1: Static responses of asymmetric laminated [0°/90°] clamped-free beamunder transverse harmonic force (L/h=50)

International Science and Technology Journal
المجلة الدولية للعلوم والتقنية

Volume 36 العدد
Part 1 المجلد

http://www.doi.org/10.62341/hama1117

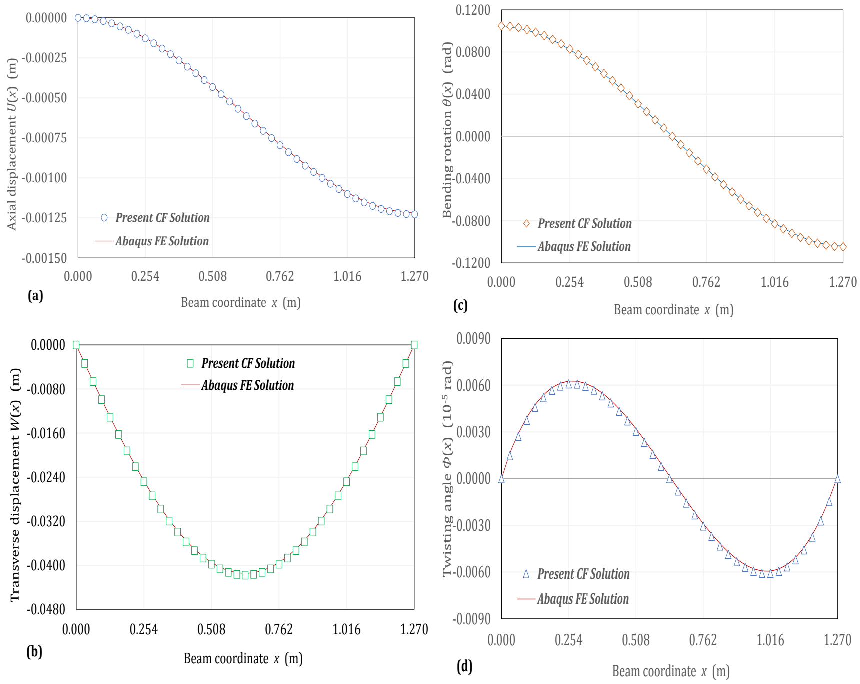

Figure 2: Static responses of asymmetric laminated [0º/90º] simply supported beamunder transverse harmonic force (*L*/*h*=50)

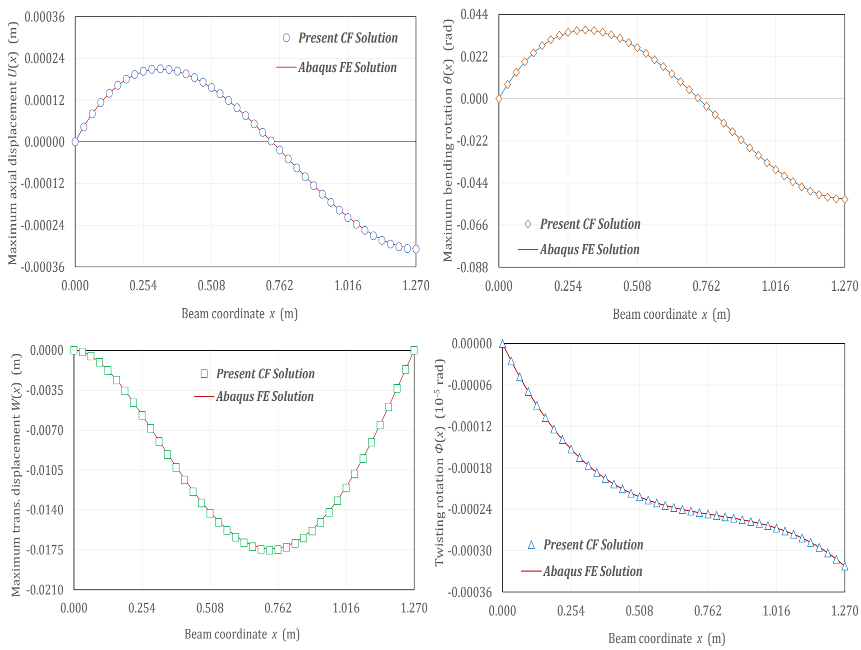

Figure 3: Static responses of asymmetric laminated [0º/90º] clamped-pinned beamunder transverse harmonic force (*L*/*h*=50)

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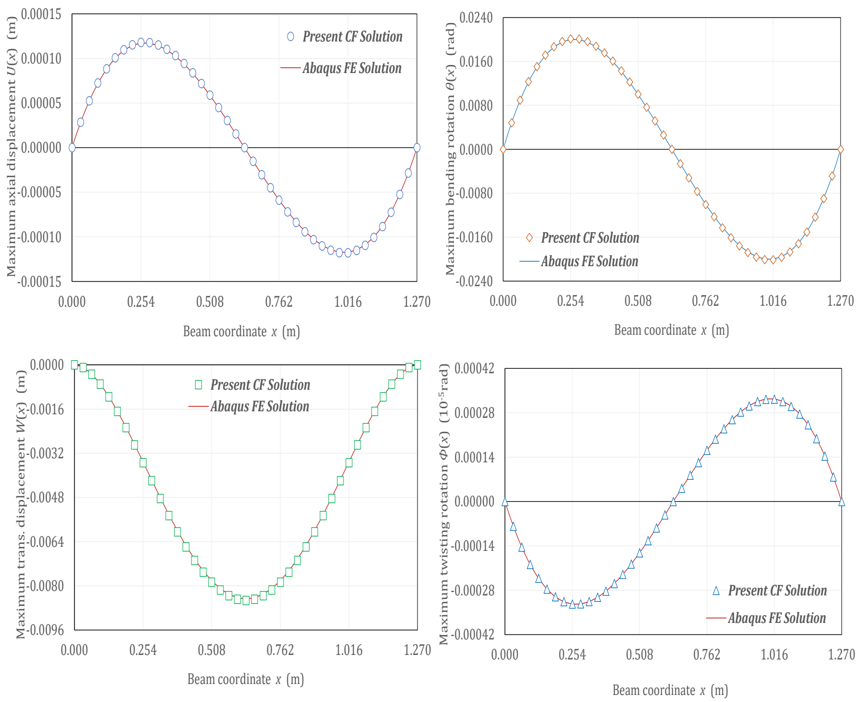

Figure 4: Static responses of antisymmetric laminated [0º/90º] clamped-clamped beamunder transverse harmonic force (*L*/*h*=50)

2.1.2 Fiber Orientation Effects on Static Response

The influence of the fiber angle (β) on the non-dimensional mid-span static displacements(W_d) of asymmetric laminated $[0^o/\beta^o]$ beams, subjected to transverse harmonic force, is presented in Figure (5) for various span-to-thickness ratios (*L*/*h*=5,10,50) under clamped-free (*CF*), simply-supported (*SS*), and clamped-clamped (*CC*) boundary conditions. A comparison with Nguyen et al. (2018) shows that the non-dimensional mid-span static results for asymmetric [0°/*β*°] laminated beams from the closed-form solution match their results precisely. This highlights the ability of the analytical approach developed by Nagiar and Hjaji (2024) to accurately capture the quasi-static behavior of these laminated beams. In other words, Figure (5) illustrate that the quasi-static coupled extensional-flexural-torsional responses, based on the closed-form solutions in (Nagiar and Hjaji 2024), align perfectly with the corresponding results from Nguyen et al. (2018), indicating excellent agreement.

International Science and Technology Journal
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Volume 36 العدد
Part 1 المجلد

http://www.doi.org/10.62341/hama1117

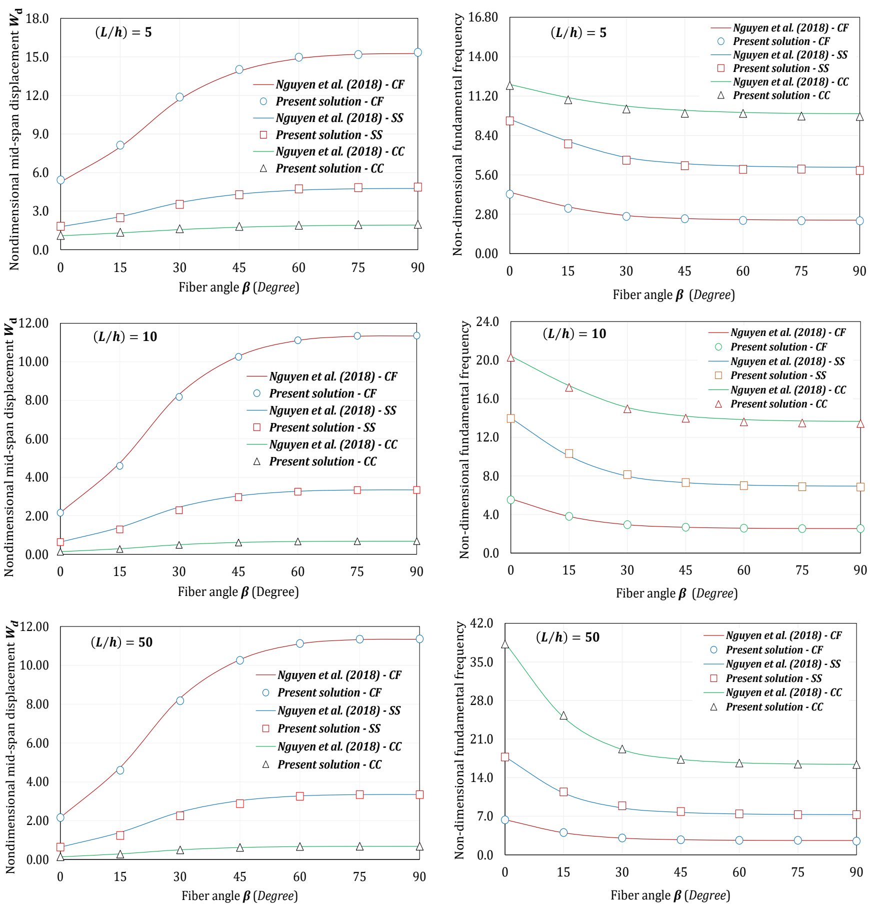

Figure 5: Effects of fiber angle on non-dimensional mid-span static displacement (W_d) and fundamental frequency $(W_f)$for laminated $[0^o/\beta^o]$beams with various boundary conditions

2.1.3 Fiber Orientation Effect on Natural Frequencies

For the sake of comparison, an asymmetric laminated $[0^o/\beta^o]$beam with the following composite material: $E_{11} = 40.0GPa,\ E_{22} = 1.0GPa,\ G_{12} = G_{13} = 0.6E_{22},\ G_{23} = 0.5E_{22},\ \upsilon_{12} = 0.25,$ and $\rho = 1389.23kg/m^3$was used by Nguyen et al. (2018) to investigate the effects of fiber angles (β^o)on the non-dimensional fundamental frequency $(W_f) = \omega L^2\sqrt{\rho/E_2h^2}$,of asymmetric laminated beams for clamped-free (*CF*), simply-supported (*SS*), and clamped-clamped(*CC*) boundary conditions and for different span-to-thickness (L/h) ratios(5, 10, and 50) as illustrated in Figure (6). It is observed that as span-to-thickness ratio (L/h) increase, the dimensionless fundamental natural frequencies (W_f) also increase.

http://www.doi.org/10.62341/hama1117

Based on the results given in graphical forms, the composite beams with a fiber orientation $\beta = 0^o$ yield the highest fundamental natural frequencies values for all three boundary conditions. In other words, the composite beams with fiber orientation ($0°/\beta = 0°$) have the highest fundamental natural frequencies compared to other fiber orientations($0^o < \beta \leq 90^o$). As fibers are aligned in the longitudinal direction ($\beta = 0°$), making the composite beam stiffer. The effect of fiber orientation (β) becomes more pronounced on the natural fundamental frequencies for fiber angles up to approximately 60° for all boundary conditions. Additionally, it is seen that, the clamped-free (*CF*) beams exhibit the highest natural frequencies while the clamped-clamped (*CC*) beams demonstrate the lowest values. Moreover, it is obvious that the results based on the closed-form solutions provided by Nagiar and Hjaji (2024) are very close to the Nguyen et al. (2018). These results establish the efficiency and accuracy of the closed-from solutions to predict the natural frequencies of the given laminated beams from the steady state dynamic analyses.

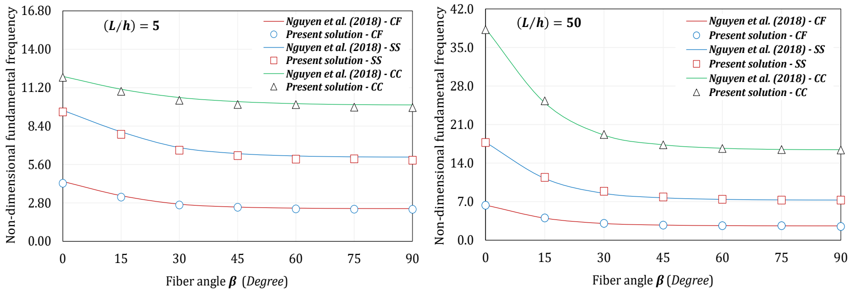

Figure 6: Fiber angle Effects on non-dimensional fundamental frequency (W_f) for laminated $[0^o/\beta^o]$ beams with various boundary conditions

2.1.4 Dynamic Analysis

The dynamic response results for the axial displacement $\overline{U}(x)$, transverse displacement $\overline{W}(x)$, bending rotation $\overline{\theta}(x)$ and torsional rotation $\overline{\Phi}(x)$ plotted against the composite beam axis x having clamped-free (*CF*), simply-supported (*SS*), clamped-clamped (*CC*), and clamped-pinned(*CP*) boundary conditions are shown in Figures (7-10), respectively. These figures demonstrate the steady state dynamic response for exciting frequency $\Omega = 62.8 rad/sec$ and for span-to thickness $L/h = 50$. For comparison, the results from the present closed-form solution and the Abaqus finite shell element solution are superimposed on the same diagrams. It is obvious that

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the dynamic results from both solutions align perfectly. Then, the steady state dynamic results obtained from closed-form solutions are in excellent agreement with the Abaqus finite element results.

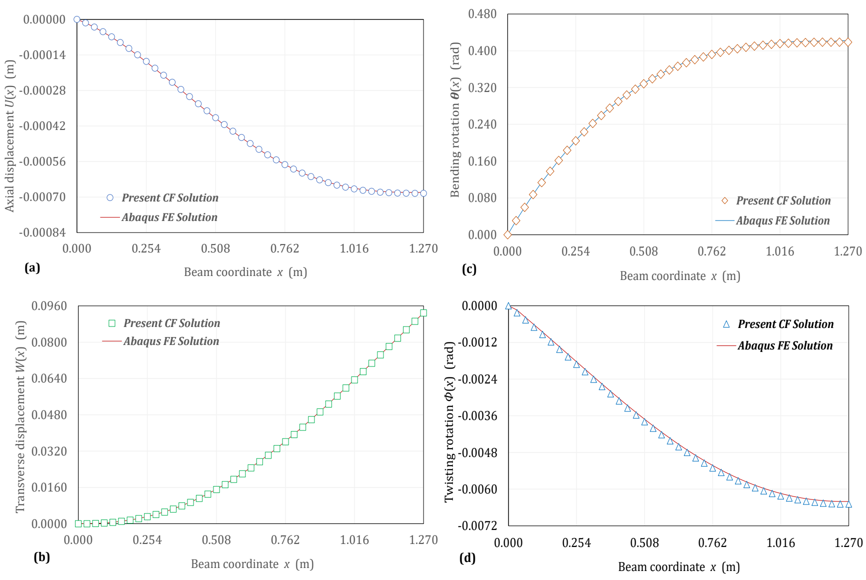

Figure 7: Dynamic response of antisymmetric (0º/90º) laminated CF-beam under harmonic bending forces with$\Omega = 62.8 rad/sec$ (L/h=50)

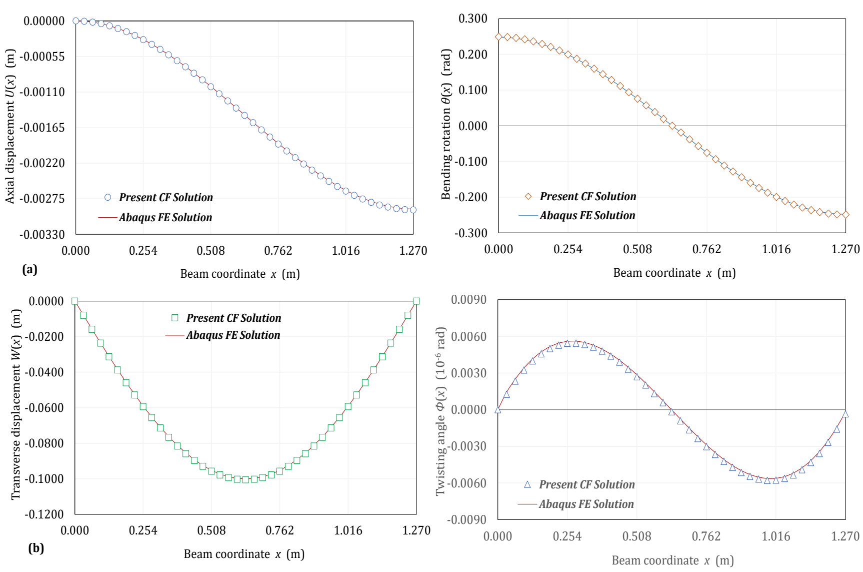

Figure 8: Dynamic response of asymmetric (0º/90º) laminated SS-beam under harmonic bending forces with$\Omega = 62.8 rad/sec$ (L/h=50)

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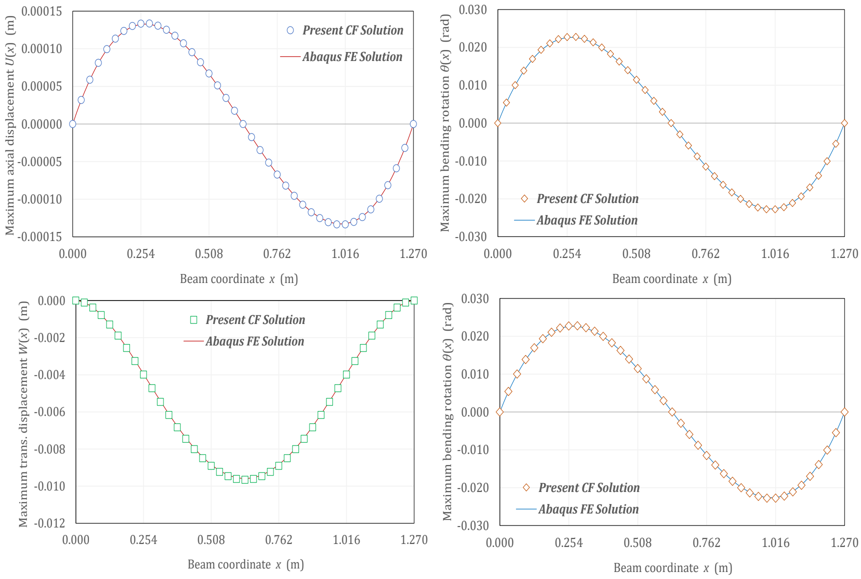

Figure 9: Dynamic response of asymmetric (0º/90º) laminated CC-beam under harmonic bending forces with$\Omega = 62.8 rad/sec$ (L/h=50)

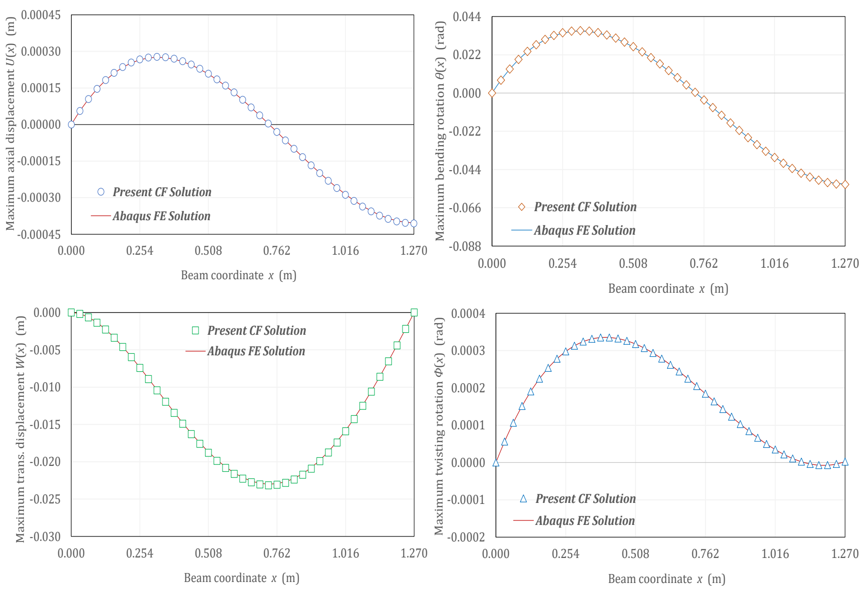

Figure 10: Dynamic response of antisymmetric (0º/90º) laminated CP-beam under harmonic bending forces with$\Omega = 62.8 rad/sec$ (L/h=50)

2.2 Example (2): Validation of Natural Frequencies

In this example, the accuracy of the present closed-form solutions are evaluated for steady state dynamic analysis of asymmetric laminated beam. For this purpose, a graphite-epoxy composite beam

http://www.doi.org/10.62341/hama1117

having four-layered asymmetric $(30^o/50^o/30^o/50^o)$laminate is considered. The asymmetric laminated composite beam with rectangular cross-section, width $b = 25.40mm$, thickness $h = 25.40mm$while the beam span$L = 0.381m$is subjected to uniformly distributed harmonic transverse force $q_z(x,t) = 8.0e^{i\Omega t}\, kN/m$ and distributed harmonic bending moment $m_x(x,t) = 6.0e^{i\Omega t}\, kN/m$. The composite material properties of the composite beam are:$E_{11} = 144.8Pa$, $E_{22} = 9.65GPa$, $G_{12} = G_{13} = 4.14GPa$,$G_{23} = 3.45Pa$, $\upsilon_{12} = 0.3$, and $\rho = 1389.23kg/m^3$. This example is frequently presented by various researchers (e.g., Jun 2008) to validate the present closed-form solution in comparison to other methods available in the literature for performing steady-state dynamic analysis, with the goal of predicting the natural coupled frequencies. It focuses on the extension-bending-shear-torsion coupled response of the given asymmetric laminated beams having different boundary conditions.

2.2.1 Natural frequencies

The steady state dynamic analyses of asymmetric $(30^o/50^o/30^o/50^o)$ laminated beams under the given distributed harmonic excitations are investigated in order to extract the coupled natural frequencies. The first five natural frequencies associated with the coupledextension-bending-shear-torsion responses are predicted from the steady state dynamic analyses when the exciting frequency $f$of the given harmonic forces and moments is varied from nearly zero to 3000*Hz*. Figures (11-13) demonstrates the variation of axial displacement $\overline{U}(x)$,transverse displacement $\overline{W}(x)$, bending rotation $\overline{\theta}(x)$and torsional twisting $\overline{\Phi}(x)$ at the beam mid-span $(i.e., x = L/2)$ of the composite laminated beams having clamped-free, clamped-pinned and clamped-clamped boundary conditions with the exciting frequency f (in *Hz*).Peaks in the diagrams represent resonance and serve as indicators of the natural frequencies of the given composite beams. Thus, the first five natural coupled frequencies extracted at the peaks of Figures (11-13) are provided in Table (2) for the asymmetric composite $(30^o/50^o/30^o/50^o)$ laminated beams having different boundary conditions.

2.2.2 Steady state mode shapes

Figures (11-13) demonstrate the first five steady state mode shapes for normalized axial displacement $(\overline{U}/\overline{U}_{max})$, transverse

http://www.doi.org/10.62341/hama1117

displacements $(\overline{W}/\overline{W}_{max})$, bending rotation $(\overline{\theta}/\overline{\theta}_{max})$and torsional angle $(\overline{\Phi}/\overline{\Phi}_{max})$ of asymmetric $(30^o/50^o/30^o/50^o)$laminated beamshaving clamped-free, clamped-pinned and clamped-clamped boundary conditions, respectively.

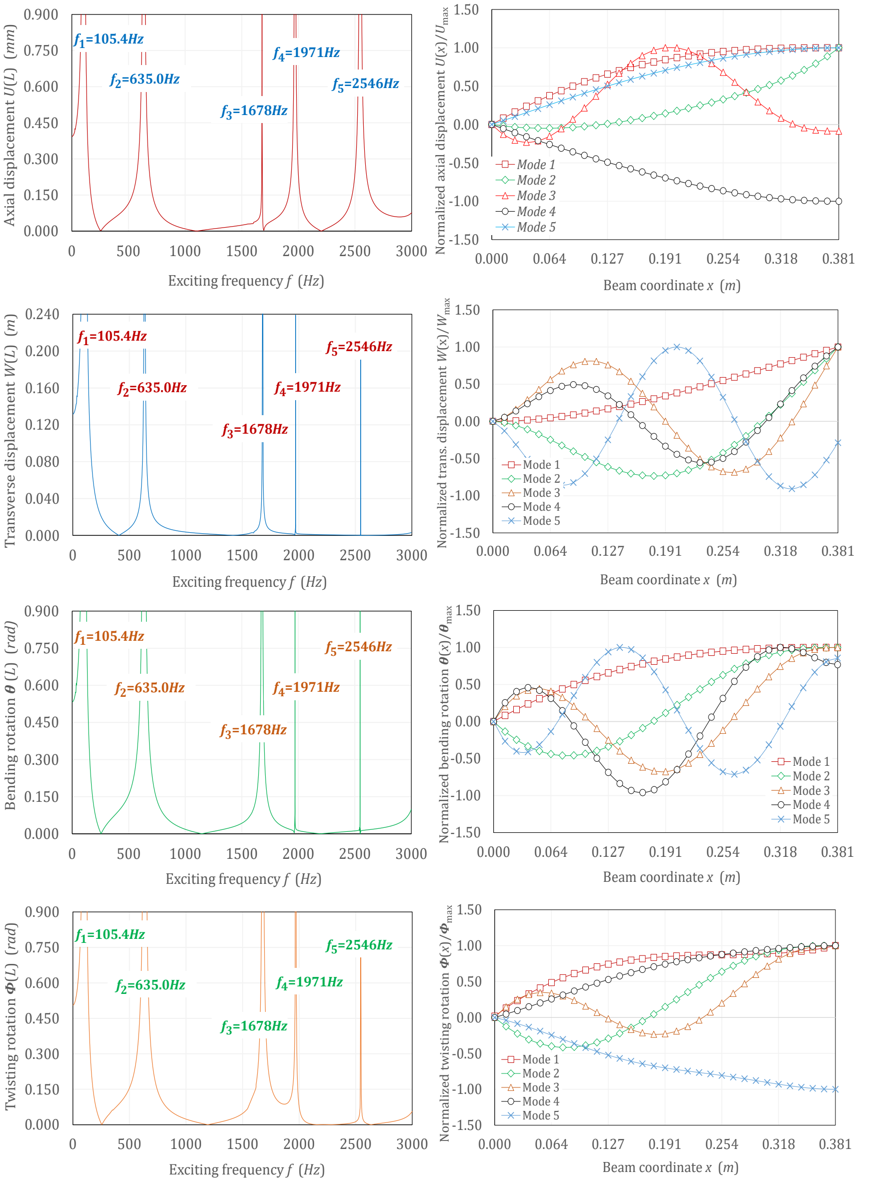

Figure 11: The natural frequencies of asymmetric (30º/50º/30º/50º) laminated CF-beam under distributed harmonic bending forces

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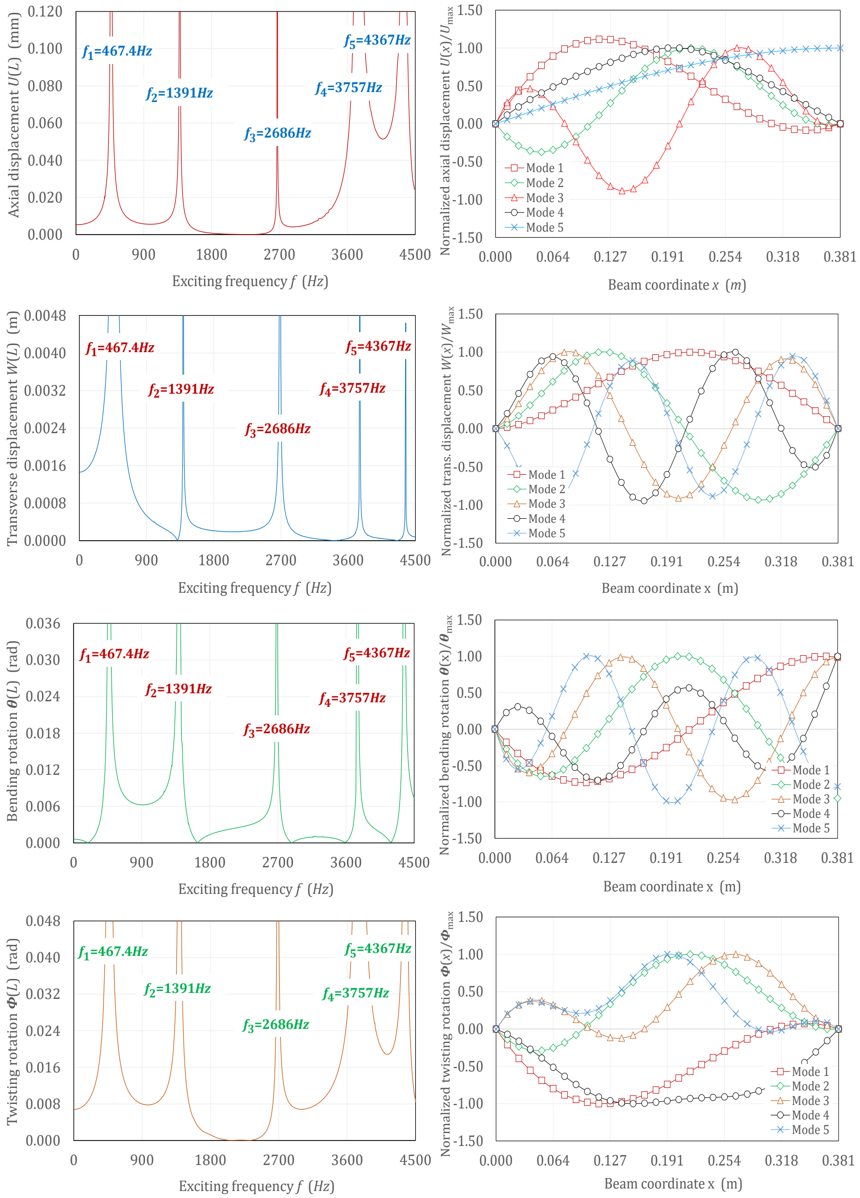

Figure 12: The natural frequencies of asymmetric (30º/50º/30º/50º) laminated CP-beam under distributed harmonic bending forces

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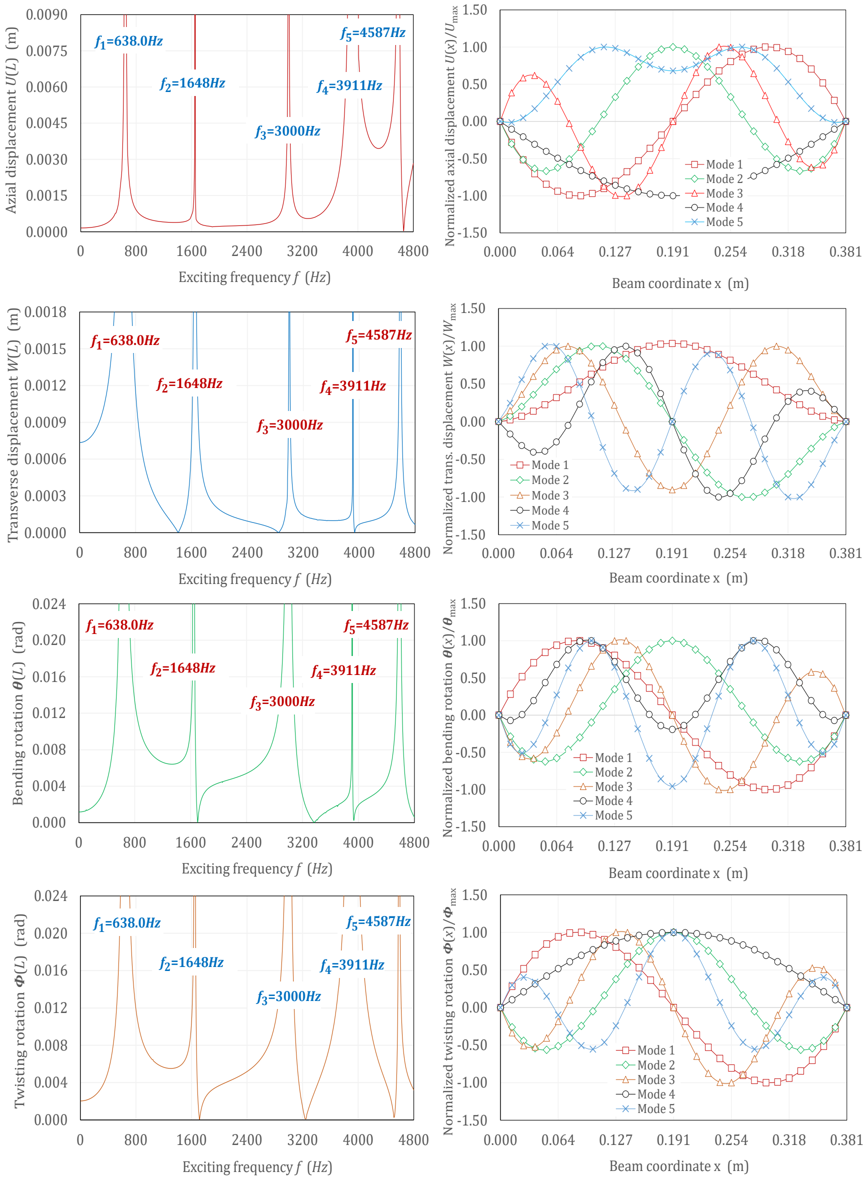

Figure 13: The natural frequencies and modes of asymmetric laminated (30°/50°/30°/50°) CC-beam under distributed harmonic bending forces

Table (2) provides the first five natural frequencies for the extension-bending-shear-torsion coupled dynamic responses of asymmetric (30°/50°/30°/50°) laminated beams under various boundary conditions, including clamped-free (*CF*), simply-supported (*SS*), clamped-pinned (*CP*), and clamped-clamped (*CC*). The natural frequencies obtained from the present closed-form

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solutions are compared with those from Jun et al. (2008). The results clearly show that the results based on the closed-form solutions closely match the reference results. Therefore, the closed-form solutions accurately capture the eigen-frequencies of the asymmetric laminated beams for the specified boundary conditions.

Table (2): The first five natural frequencies for asymmetric laminated (30º/50º/30º/50º) beams with various boundary conditions

Boundary conditions	Mode number	Natural frequencies in *Hz*		%Error
		Present	Jun et al. [6]	
Clamped-free (*CF*)	1	105.4	105.3	0.09
	2	635.0	635.0	0.00
	3	1678.0	1678.9	-0.05
	4	1971.0	1970.6	0.02
	5	2546.0	2546.2	-0.01
Simply supported (*SS*)	1	353.80	353.70	0.03
	2	1114.0	1114.3	-0.03
	3	2434.0	2434.2	-0.01
	4	3450.0	3450.3	-0.01
	5	4265.0	4264.5	0.01
Clamped-pinned (*CP*)	1	467.40	467.30	0.02
	2	1391.0	1391.3	-0.02
	3	2686.0	2685.5	0.02
	4	3757.0	3757.0	0.00
	5	4367.0	4366.8	0.00
Clamped-clamped (*CC*)	1	638.00	637.90	0.02
	2	1648.0	1647.8	0.01
	3	3000.0	3000.0	0.00
	4	3911.0	3911.0	0.00
	5	4587.0	4587.3	-0.01

2.3. Example (3): Validations of Static and Dynamic Analyses

A graphite-polyester asymmetric$(0^o/90^o/0^o/90^o)$laminated clamped-free beam subjected to uniformly distributed harmonic transverse force $q_z(x,t) = 4.0e^{i\Omega t}\ kN/m$ and distributed twisting moment $m_{xy}(x,t) = 2.0e^{i\Omega t}\ kNm/m$ is considered for static and dynamic analyses. The geometric properties of the composite beam are: length $L = 10h$, width $b = 40mm$, and thickness $h = 40mm$, while the composite material properties are: $E_{11} = 221.0Pa$, $E_{22} = 6.90GPa$, $G_{12} = 4.80GPa$,$G_{13} = 4.14GPa$, $G_{23} = 3.45Pa$, $\upsilon_{12} = 0.3$, and $\rho = 1550.1\ kg/m^3$.

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The quasi-static and steady state dynamic analyses of the extension-bending-shear-torsion coupled responses of the four-layered asymmetric $(0^o/90^o/0^o/90^o)$laminated beamunder the given distributed bending and twisting loads are investigated. The numerical results obtained from the exact formulation developed by the authors Nagiar and Hjaji (2024) are compared to the corresponding results based on the Abaqus finite shell element model solution.

2.3.1 Quasi-Static Solution

Based on the exact closed-form solutions, the quasi-static results obtained using very low exciting frequency (i.e., $\Omega \approx 0.01f_1 = 2.99Hz$, where $f_1 = 299.0Hz$) for extensional displacement $\overline{U}(x)$,transverse displacement $\overline{W}(x)$, bending rotation $\overline{\theta}(x)$, and twisting angle $\overline{\Phi}(x)$ are plotted against beam coordinate *x* as shown in Figure (14) for asymmetric laminated clamped-free beam, respectively.

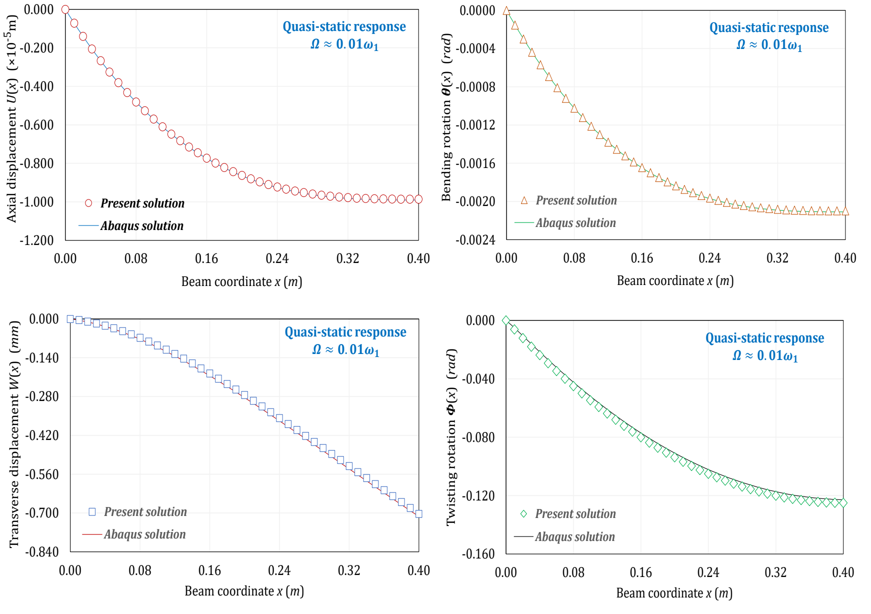

Figure 14: Static responses of asymmetric (0º/90º/0º/90º) laminated CF-beam under distributed harmonic bending force and twisting moment with exciting frequency $\Omega \approx 0.01\omega_1$

The static results obtained from the closed-form solution and Abaqus finite shell solution are plotted on the same diagrams for the

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sake of comparison. The graphical data indicate that the static response results from the present formulation closely match those of the Abaqus finite shell model. Again, this suggests that, the present solution effectively captures the static response of the composite laminated beam.

2.3.2 Steady state dynamic solution

Under the given harmonic excitations with exciting frequency $f = 400Hz$, the dynamic responses for extensional displacement $\overline{U}(x)$, transverse displacement $\overline{W}(x)$, bending rotation $\overline{\theta}(x)$, and twisting angle $\overline{\Phi}(x)$ versus the beam axis x of asymmetric laminated clamped-free beam are shown in Figure (15), respectively. As illustrated in Figure (15), there is an excellent agreement between the predictions of the dynamic response from the closed-form solution and Abaqus finite element solution.

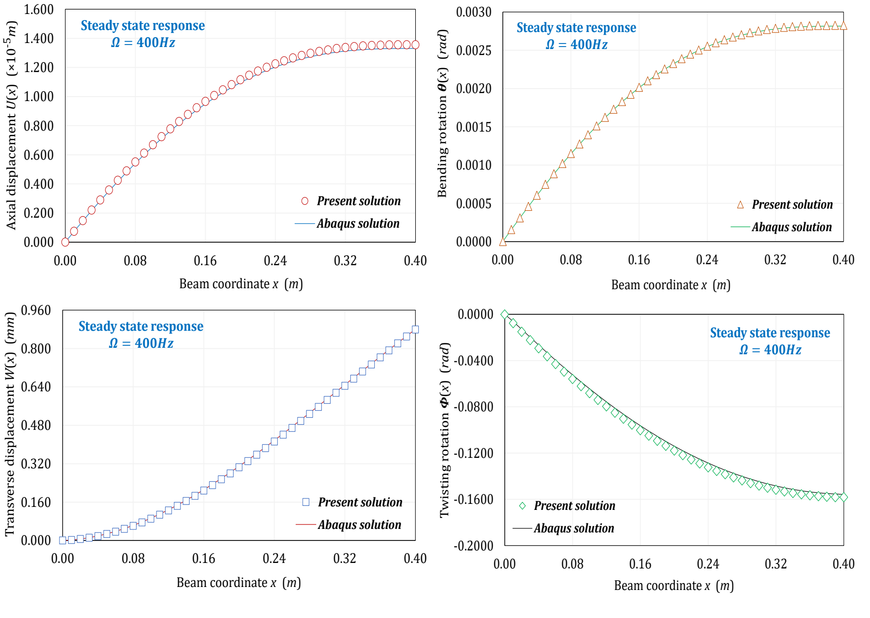

Figure (15): Dynamic responses for asymmetric (0º/90º/0º/90º) laminated CF-beam under distributed harmonic bending force and twisting moment with exciting frequency$\Omega = 400Hz$

3. Conclusion

1. The closed-form solutions for the coupled extension-bending-shear-torsion field equations were previously obtained in Nagiar

http://www.doi.org/10.62341/hama1117

and Hjaji (2024) for asymmetric laminated composite beams with clamped-free, simply-supported, clamped-clamped, and clamped-pinned boundary conditions.
2. The exact closed-form solutions effectively capture the quasi-static at low exciting frequencies and steady-state dynamic responses of asymmetric laminated composite beams subjected to general harmonic excitations.
3. The closed-form solutions successfully extract the coupled natural frequencies and steady-state mode shapes.
4. A comparison of the present results based on the closed-form solutions with established Abaqus finite element solutions and exact solutions available in the literature confirms the validity and accuracy of the present closed-form solutions.
5. It is observed that the natural frequencies of asymmetric laminated beams generally decrease as the fiber orientation angle increases.
6. The results observed that as the span-to-thickness ratio (L/h) increases, the non-dimensional fundamental natural frequencies also increase.
7. The research highlights the applicability of the developed solutions to composite beams in engineering fields such as aerospace, automotive, and civil engineering, where complex dynamic loads are present.
8. The proposed closed-form solutions are capable of accurately predicting coupled dynamic responses, making them useful for future studies on more complex beam configurations, nonlinear effects, and various loading conditions.
9. The results provide a strong foundation for further exploration into laminated composite structures subjected to dynamic loads, contributing valuable insights to the field of composite beam design.

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